1. Geometrical Optics and Optical Instruments

1. Cardinal Points of an Optical system: -

In case of refraction through a thin lens, the thickness of the lens has been neglected in calculating the various formulae. It is immaterial from which point of the lens the distances are measured. But in case of a thick lens or a in a combination of two lenses separated by a finite distance, we cannot proceed with this assumption. Moreover, the method of evaluating the distance of the image considering refraction at each surface of a lens successively is extremely tedious.

To overcome this difficulty, Gauss in 1841 proved that any number of co-axial refracting systems can be applied as one unit and the simple formulae for thin lenses can be applied, provided the distances are measured from two theoretical parallel planes, fixed with reference to the refracting system.

The points of intersection of these planes with the axis are called the principal or Gauss points.

Actually, there are six points in all, the two principal foci (focal points), the two principal points and the two nodal points. These six points are known as **cardinal points** of an optical system.

i) Principal Foci and Focal Planes:

Consider an optical system consisting of a thick lens or a number of coaxial lenses either in contact or separated by an appreciable distance and having its axis AA′.

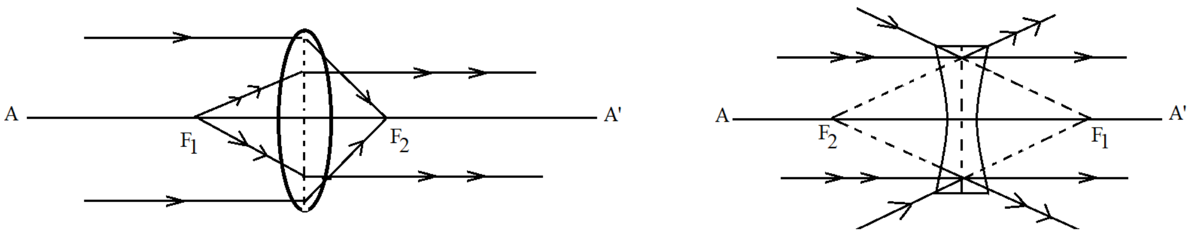


Fig. (i) Converging System

Fig. (ii) Diverging System

A set of rays incident on the system parallel to the axis, after refraction converge to converging system or appear to diverge from diverging system a point F_2 on the axis. This point F_2 is called second principal focus and is the position of the image corresponding to the axial point object at infinity ($u=\infty$) as shown in Fig. (i).

Similarly, if the rays starting from converging system or directed towards diverging system an axial point F_1 after refraction through the systems becomes parallel to the axis, then this point F_1 is called the first principal focus and corresponds to the image distance at infinity ($v=\infty$) as shown in Fig. (ii).

These two points F_1 and F_2 are called the principal foci or focal points and planes passing through the principal foci perpendicular to the axis are called focal planes. The main property of the focal planes is that the rays starting from a point in the focal plane in the object space correspond to a set of conjugate parallel rays in the image space. Similarly, a set of parallel rays in the object space corresponds to a set of rays intersecting at a point in the focal plane in the image space.

ii) Principal Points and Principal Planes:

There are two principal planes and two principal points. The first principal plane in the object space is the locus of the points of interaction of the emergent rays in the image space parallel to the axis and their conjugate incident rays in the object space. The second principal planes in the image space is the locus of the points of interaction of the incident rays in the object space parallel to the axis and their conjugate emergent rays in the image space.

Consider a thick lens or a co-axial refracting system having its principal foci F_1 and F_2 as shown in fig(i).

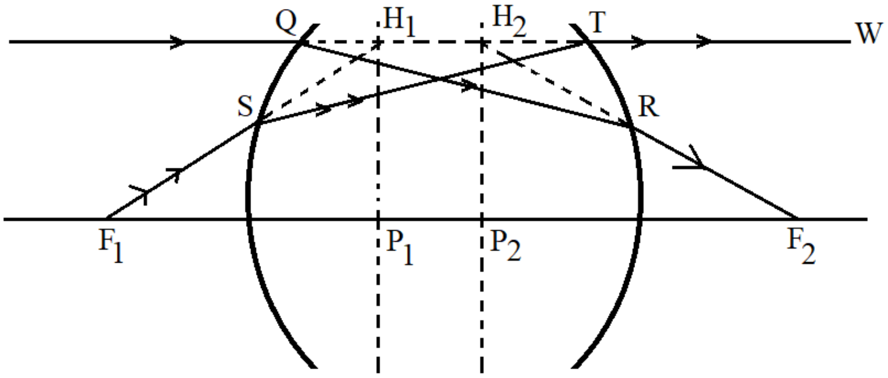


Fig. (i)

The ray incident at the point Q and parallel to the axis, after refraction through the lens takes the direction RF_2 passing through the second principal focus F_2. The incident and the emergent rays, when produced intersect at H_2. A plane passing through H_2 and perpendicular to the axis is termed as second principal plane of the lens. Its point of intersection P_2 with the axis is called the second principal point.

Consider another ray F_1S passing through the first principal focus F_1 such that after refraction it emerges along TW parallel to the axis at the same height as that of the ray incident at Q. The rays F_1S and TW when produced intersect at H_1. A plane perpendicular to the axis and passing through H_1 is called the first principal plane and its point of interaction P_1 with the axis is called the first principal point.

It is clear from the figure that the two incident rays are directed towards H_1 and after refraction appear to come from H_2. Therefore, H_2 is the image of H_1. Thus, H_1 and H_2 are the conjugate points and the planes H_1P and H_2P are a pair of conjugate planes. Further $H_2P_2 = H_1P_1$. The lateral magnification of the planes +1.

iii). Nodal Points:

Nodal points are defined as a pair of conjugate points on the axis having unit positive angular magnification. This means that a ray of light directed towards one of these points, after refraction through the optical system appears to proceed from the second point in a parallel direction as shown in fig. (i).

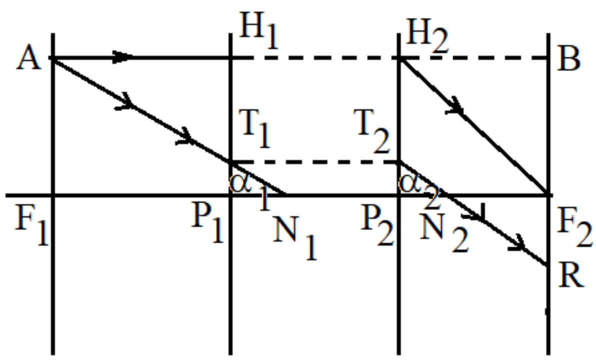


Fig. (i)

Let H_1P_1 and H_2P_2 be first and second principal planes of an optical system and let AF_1 and BF_2 be its first and second focal planes respectively. Consider a point A situated on the first focal plane. From A draw a ray AH_1 parallel to the axis. The conjugate ray will proceed from H_2, a point in the second principal plane such that $H_2P_2 = H_1P_1$ and will pass through the second focus F_2.

Take another ray AT_1 parallel to the emergent ray H_2F_2 and striking the first principal plane at T_1. It will emerge out from T_2, a point on the second plane such that $T_2P_2=T_1P_1$ and will proceed parallel to the ray H_2F_2 as the two rays originate at A, a point on first focal plane. Then the points of interaction of the incident ray AT_1 and conjugate emergent ray T_2R with the axis give the positions of two nodal points. It is clear that the two points N_1 and N_2 are a pair of conjugate points and incident ray AN_1 is parallel to the conjugate emergent ray T_2R. Moreover, $tan\alpha_1 = tan\alpha_2$.

$$\frac{tan\alpha_2}{tan\alpha_1} = 1$$

In right angled triangle, $\Delta T_1P_1N_1 \; and \; T_2P_2N_2$,

$T_1P_1 = T_2P_2$ and $< T_1N_1P_1 = < T_2N_2P_2 = \alpha$

Here two triangles are congruent, $\therefore P_1N_1 = P_2N_2$

Adding N_1P_2 to both sides

$$\therefore P_1N_1 + N_1P_2 = P_2N_2 + N_1P_2$$

$$P_1P_2 = N_1N_2$$

The distance between the principal points N_1 and N_2 is equal to distance between principal points P_1 and P_2.

Now consider the two right angled triangle ΔAF_1N_1 and $H_2P_2F_2$.

$AF_1 = H_2P_2$ and $\angle AN_1F_1 = \angle H_2F_2P_2$

Hence two triangles are congruent.

$\therefore F_1N_1 = P_2F_2$

$F_1P_1+P_1N_1 = P_2F_2$

$P_1N_1 = P_2F_2 - F_1P_1$

Also $P_1N_1 = P_2N_2$

$P_2F_2 = +f_2$ and $P_1F_1 = -f_1$

$P_1N_1 = P_2N_2 = (f_1+f_2)$

If the medium on both the sides of the system is optically similar,

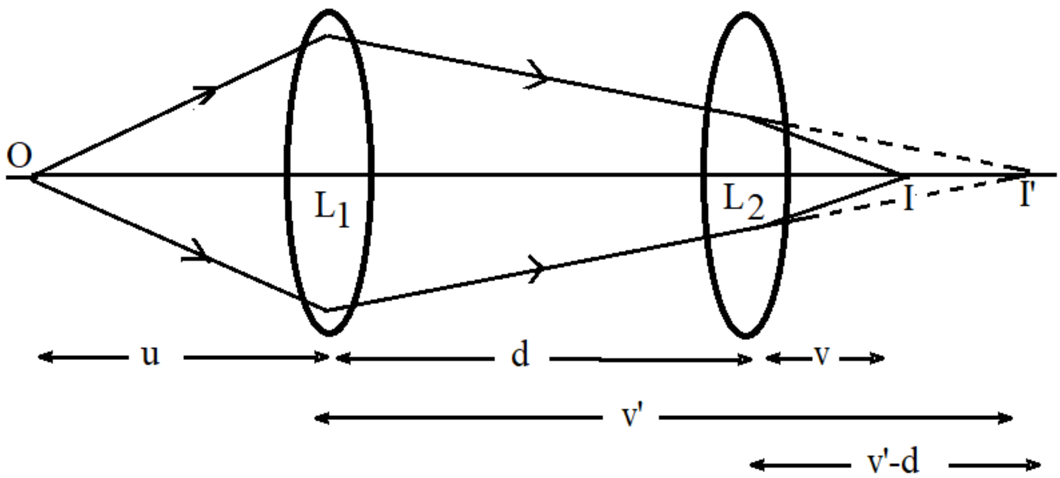
$$f_2 = -f_1$$

$$\therefore P_1N_1 = P_2N_2 = 0$$

It means the principal points coincide with the nodal points when the optical system is situated in the same medium.

The planes passing through the nodal points and perpendicular to the axis are termed as nodal planes.

2. Coaxial lens system - equivalent focal length and cardinal points.



Consider two thin lenses L_1 and L_2 of focal lengths f_1 and f_2 respectively. A point object O is placed at a distance u from the first lens and final image is formed at I. The first image due to first lens is formed at I′.

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1}$$

$$\frac{1}{v'} = \frac{1}{f_1} + \frac{1}{u} = \frac{u + f_1}{f_1 u}$$

$$v' = \frac{u f_1}{u + f_1} - - - - - - -(1)$$

The image behaves as an object for the second lens. Therefore, the object distance for second lens is v′-d and final image is formed at I.

$$\frac{1}{v} - \frac{1}{v' - d} = \frac{1}{f_2}$$

$$\frac{1}{v' - d} = \frac{1}{v} - \frac{1}{f_2} = \frac{f_2 - v}{v f_2}$$

$$\therefore \mathrm{v}' - d = \frac{\mathrm{v}f_2}{f_2 - \mathrm{v}}$$

Substituting value of v′ from eq^{n} (1)

$$\frac{uf_1}{u + f_1} - d = \frac{\mathrm{v}f_2}{f_2 - \mathrm{v}}$$

$$\frac{uf_1 - d(u + f_1)}{u + f_1} = \frac{\mathrm{v}f_2}{f_2 - \mathrm{v}}$$

$$(f_2 - \mathrm{v})[\, uf_1 - d(u + f_1)] = \mathrm{v}f_2[u + f_1]$$

$$uf_1f_2 - f_2d(u + f_1) - \mathrm{v}uf_1 + \mathrm{v}d(u + f_1) = \mathrm{v}f_2u + \mathrm{v}f_1f_2$$

$$uf_1f_2 - udf_2 - df_1f_2 - \mathrm{v}uf_1 + \mathrm{vdu} + \mathrm{vd}f_1 = \mathrm{v}f_2u + \mathrm{v}f_1f_2$$

$$u\mathrm{v}(\mathrm{d} - f_1 - f_2) + u(f_1f_2 - df_2) + \mathrm{v}(\mathrm{d}f_1 - f_1f_2) - df_1f_2 = 0 - - - - (2)$$

This equation can be written in the form

$$u\mathrm{v}A + uB + \mathrm{vC} + D = 0 - - - - - - (3)$$

Where A=$(\mathrm{d} - f_1 - f_2)$, B=$(f_1f_2 - df_2)$, C= $(\mathrm{d}f_1 - f_1f_2)$ and D =$-df_1f_2$.

A, B, C are coefficients.

$$\therefore u\mathrm{v} + \mathrm{u}\frac{B}{A} + \mathrm{v}\frac{C}{A} + \frac{D}{A} = 0 - - - - - - (4)$$

Let *f* be the focal length of equivalent lens and reduced object distance U=u-α and reduced image distance V= v-β.

Let α represents the distance of the first lens from the first principal plane and β represents the distance of the second lens from the second principal plane.

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{f}$$

$$\frac{1}{\mathrm{v} - \beta} - \frac{1}{\mathrm{u} - \alpha} = \frac{1}{f}$$

$$\frac{(u - \alpha) - (\mathrm{v} - \beta)}{(\mathrm{v} - \beta)(u - \alpha)} = \frac{1}{f}$$

$$f(u - \alpha) - f(\mathrm{v} - \beta) = (\mathrm{v} - \beta)(u - \alpha)$$

$$fu - f\alpha - f\mathrm{v} + \mathrm{f\beta} = \mathrm{uv} - \mathrm{v\alpha} - \mathrm{u\beta} + \mathrm{\alpha\beta}$$

$$u\mathrm{v} + \mathrm{u}(-\mathrm{f} - \beta) + \mathrm{v}(\mathrm{f} - \alpha) + (\alpha\beta - \beta\mathrm{f} + \alpha\mathrm{f}) = 0 - - - - (5)$$

Comparing equation (4) and (5)

$$-\beta - f = \frac{B}{A} - - - (6)$$

$$-\alpha + f = \frac{C}{A} - - - - (7)$$

$$\alpha\beta - \beta\mathrm{f} + \alpha\mathrm{f} = \frac{D}{A} - - - - (8)$$

multiplying equation (6) and (7)

$$(-\beta - f)(-\alpha + f) = \frac{B}{A}.\frac{C}{A}$$

$$\alpha\beta - \beta f + \alpha f - f^2 = \frac{BC}{A^2} - - - - (9)$$

Subtracting equation (9) from equation (8)

$$f^2 = \frac{D}{A} - \frac{BC}{A^2} = \frac{DA - BC}{A^2} - - - - - - (10)$$

Substituting the values of the coefficients A, B, C and D.

$$f^2 = \frac{(-df_1f_2)(\mathrm{d} - f_1 - f_2) - (f_1f_2 - df_2)(\mathrm{d}f_1 - f_1f_2)}{(\mathrm{d} - f_1 - f_2)^2}$$

$$f^2 = \frac{-d^2f_1f_2 + df_1{}^2f_2 + df_1f_2{}^2 - df_1{}^2f_2 + f_1^2f_2^2 + d^2f_1f_2 - df_1f_2{}^2}{(f_1 + f_2 - d)^2}$$

$$f^2 = \frac{f_1^2f_2^2}{(f_1 + f_2 - d)^2}$$

Taking square root,

$$f = \pm\frac{f_1f_2}{f_1 + f_2 - d}$$

$$\frac{1}{f} = \frac{f_1 + f_2 - d}{f_1f_2} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1f_2} - - - - - (11)$$

Also, $\frac{f_1 f_2}{f} = f_1 + f_2 - d$

From equation (7)

$$-\alpha + f = \frac{C}{A}$$

$$\alpha = -\frac{C}{A} + f = -\frac{\mathrm{d}f_1 - f_1 f_2}{\mathrm{d} - f_1 - f_2} + \frac{f_1 f_2}{f_1 + f_2 - d}$$

$$\alpha = \frac{\mathrm{d}f_1 - f_1 f_2 + f_1 f_2}{f_1 + f_2 - d} = \frac{\mathrm{d}f_1}{f_1 + f_2 - d} - - - - (12)$$

Also, $f_1 + f_2 - d = \frac{f_1 f_2}{f}$

$$\alpha = \mathrm{d}f_1 \times \frac{f}{f_1 f_2} = \frac{+df}{f_2} - - - - - (13)$$

Thus, the first principal plane is to the right of the first lens.

From equation (6),

$$-\beta - f = \frac{B}{A}$$

$$\beta = -f - \frac{B}{A} = -[\frac{fA + B}{A}]$$

$$\beta = -\left[\frac{(f_1 f_2 - df_2) + (\mathrm{d} - f_1 - f_2) \times \frac{f_1 f_2}{f_1 + f_2 - d}}{(\mathrm{d} - f_1 - f_2)}\right]$$

$$\beta = -\left[\frac{f_1 f_2 - df_2 - f_1 f_2}{(\mathrm{d} - f_1 - f_2)}\right] = \frac{df_2}{\mathrm{d} - f_1 - f_2} = \frac{-df_2}{f_1 + f_2 - d} - - - (14)$$

Also, $f_1 + f_2 - d = \frac{f_1 f_2}{f}$

$$\beta = -df_2 \times \frac{f}{f_1 f_2} = \frac{-df}{f_1} - - - - (15)$$

Thus, the second principal plane is to the left of the second lens.

From equation (13) and (15)

$$-\beta f_1 = +\alpha f_2$$

$$\frac{f_1}{f_2} = -\frac{\alpha}{\beta}$$

Special Case: when two lenses are in contact, d=0

Therefore, equation (11) becomes

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Power: Power is the reciprocal of focal length in meters.

From equation (11)

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

But $\frac{1}{f} = P, \frac{1}{f_1} = P_{1,}\ and\ \frac{1}{f_2} = P_{2,}$

$$\therefore P = P_1 + P_2 - dP_1P_2$$

Example 1: A co-axial lens system placed in air has two lenses of focal length 3F and F separated by a distance 2F. Find the positions of the cardinal points.

Sol^n: Given

f_1 = 3F, f_2 = F and d= 2F

The equivalent focal length is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$= \frac{f_2 + f_1 - d}{f_1 f_2}$$

$$\therefore f = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{3F.F}{3F + F - 2F} = \frac{3F^2}{2F} = \frac{3}{2}F$$

$$\alpha = \frac{+df}{f_2} = +\frac{2F.\frac{3}{2}F}{F} = +3F$$

Therefore, the first principal point P_1 is at a distance 3F to right of the first lens.

$$\beta = \frac{-df}{f_1} = -\frac{2F.\frac{3}{2}F}{3F} = -F$$

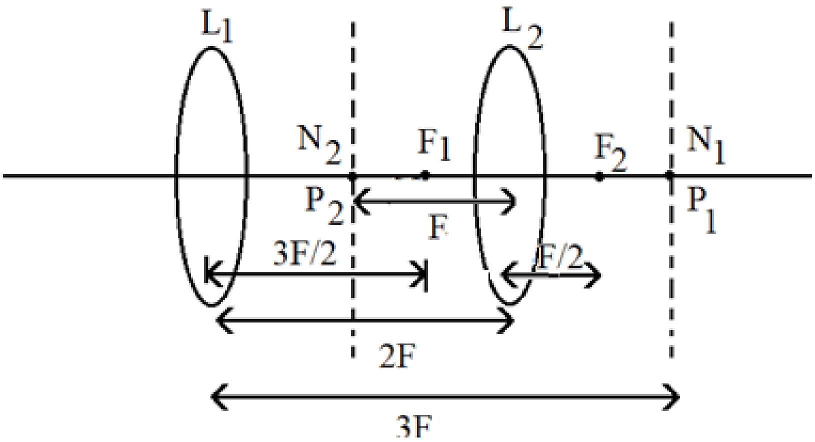
The second principal point P2 is at a distance F to left of the second lens.

The first focal point F_1 is at a distance $=f_1$-f = 3F-3F/2 = 3F-1.5F=1.5F from the first lens.

The second focal point F2 is at distance = f-f_2 = 3F/2-F = 1.5F-F = 0.5F from the second lens.

As the medium on the two sides of the lens system is the same, the nodal points N_1 and N_2 coincide with P_1 and P_2.

The position of cardinal points are as shown in figure.



Example 2. Two thin convex lenses having focal lengths 5cm and 2 cm are coaxial and separated by a distance of 3 cm. Find the equivalent focal length and position of cardinal points.

Sol^n: Given

f_1 = 5 cm, f_2 = 2cm and d= 3cm

The equivalent focal length is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$= \frac{f_2 + f_1 - d}{f_1 f_2}$$

$$\therefore f = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{5 \times 2}{5 + 2 - 3} = \frac{10}{4} = 2.5\ cm$$

$$\alpha = \frac{+df}{f_2} = +\frac{3 \times 2.5}{2} = \frac{7.5}{2} = 3.75\ cm$$

Therefore, the first principal point P_1 is at a distance 3.75 cm to right of the first lens.

$$\beta = \frac{-df}{f_1} = -\frac{3 \times 2.5}{5} = \frac{-7.5}{5} = -1.5\ cm$$

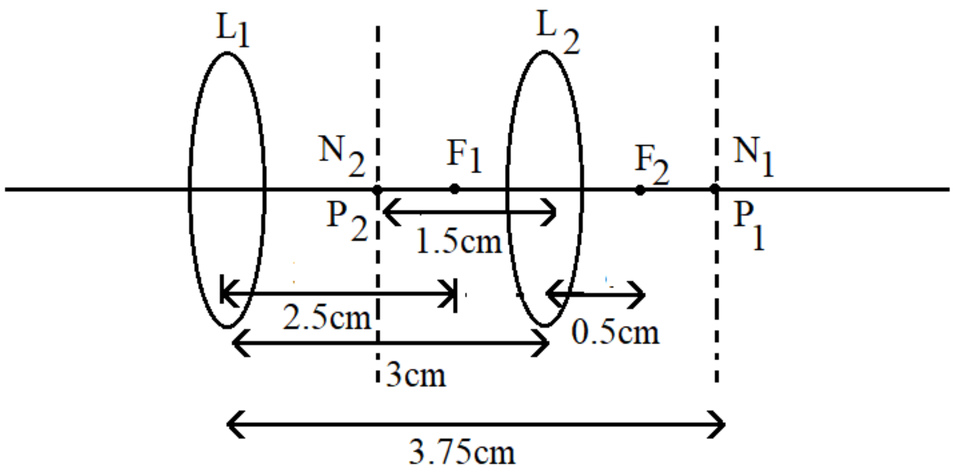
The second principal point P_2 is at a distance 1.5 cm to left of the second lens.

The first focal point F_1 is at a distance $= f_1 - f = 5 - 2.5 = 2.5$ cm from the first lens.

The second focal point F_2 is at distance $= f - f_2 = 2.5 - 2 = 0.5$cm from the second lens.

As the medium on the two sides of the lens system is the same, the nodal points N_1 and N_2 coincide with P_1 and P_2.

The position of cardinal points is as shown in figure.



3. Huygens Eyepiece and Their Cardinal Points:

Huygens eyepiece is achromatic and the spherical aberration is also eliminated. It consists of two lenses having focal lengths in the ratio 3:1 and distance between two lenses is equal to the difference in their focal lengths. The focal lengths and the position of the two lenses are such that each lens produces an equal deviation of the ray and the system is achromatic.

Suppose the field lens and the eye lens of focal lengths f_1 and f_2 are placed D cm apart. If F is the focal length of the combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{D}{f_1 f_2}$$

Differentiating

$$\frac{-dF}{F^2} = \frac{-df_1}{{f_1}^2} - \frac{df_2}{{f_2}^2} - D.\left(\frac{-df_1}{{f_1}^2}.\frac{1}{f_2} + \frac{1}{f_1}.\frac{-df_2}{{f_2}^2}\right)$$

$$\therefore \frac{-dF}{F^2} = \frac{-df_1}{{f_1}^2} - \frac{df_2}{{f_2}^2} + D.\left(\frac{df_1}{{f_1}^2 f_2} + \frac{df_2}{f_1 {f_2}^2}\right) - - - - (1)$$

As the dispersive power is defined as,

$$\omega = \frac{-dF}{F} = \frac{-df_1}{f_1} = \frac{-df_2}{f_2}$$

Equation (1) becomes,

$$\frac{\omega}{F} = \frac{\omega}{f_1} + \frac{\omega}{f_2} - D\left(\frac{\omega}{f_1 f_2} + \frac{\omega}{f_1 f_2}\right)$$

$$\frac{\omega}{F} = \frac{\omega}{f_1} + \frac{\omega}{f_2} - D\left(\frac{2\omega}{f_1 f_2}\right)$$

For achromatism, $\frac{\omega}{F} = 0$

$$\therefore \frac{\omega}{f_1} + \frac{\omega}{f_2} - D\left(\frac{2\omega}{f_1 f_2}\right) = 0$$

$$\therefore \frac{1}{f_1} + \frac{1}{f_2} - \frac{2D}{f_1 f_2} = 0$$

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{2D}{f_1 f_2}$$

$$\frac{f_1 + f_2}{f_1 f_2} = \frac{2D}{f_1 f_2}$$

$$\therefore 2D = f_1 + f_2$$

$$D = \frac{f_1 + f_2}{2} - - - - - - (2)$$

Also, for equal deviation of a ray by the two lenses, the distance between the two lenses should be equal to f_1-f_2.

Thus, to satisfy both the conditions, Huygens constructed an eyepiece consisting of two plano-convex lenses of focal lengths 3f and f placed at a distance of 2f from each other as shown in fig (a).

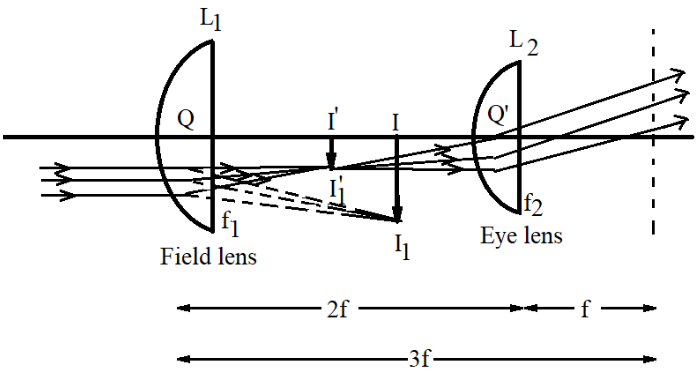


Fig. (a)

Let II_1 is the of the distant object formed by the objective in the absence of the field lens. With the field lens, the rays get refracted on passing through it and the image $I'I_1'$ is formed. This image lies at the focus of the eye lens so that the final image is seen at infinity.

The focal length of the combination is

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

$$\frac{1}{F}=\frac{1}{3f}+\frac{1}{f}-\frac{2f}{3f.f}=\frac{f+3f-2f}{3f^2}$$

$$\frac{1}{F}=\frac{2f}{3f^2}=\frac{2}{3f}$$

$$\therefore F=\frac{3f}{2}$$

The equivalent lens must be placed behind the field lens at a distance,

$$=\frac{F\times d}{f_2}=\frac{\frac{3f}{2}\times 2f}{f}=3f$$

i.e. 3f from the field lens or at a distance f behind the eye lens.

Huygens eyepiece is known as the negative eyepiece because the real inverted image formed by the objective lies behind the field lens and this image acts as a virtual object for the eye lens. This eyepiece cannot be used to examine directly an object or a real image formed by the objective. The eyepiece is used in microscopes or other optical instruments using white light only. The cross wires cannot be used in a Huygens eyepiece.

Huygens eyepiece cannot be used in telescopes and other optical instruments with which distance and angles are to be measured.

Cardinal Points of a Huygens Eyepiece:

Huygens constructed an eyepiece consisting of two plano-convex lenses of focal lengths 3f and f placed at a distance of 2f from each other.

$f_1 = 3f$, $f_2 = f$ and $d = 2f$

The equivalent focal length of the combination is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F} = \frac{1}{3f} + \frac{1}{f} - \frac{2f}{3f.f} = \frac{f + 3f - 2f}{3f^2}$$

$$\frac{1}{F} = \frac{2f}{3f^2} = \frac{2}{3f}$$

$$\therefore F = \frac{3f}{2}$$

The first principal point is at a distance α from the field lens,

$$\alpha = \frac{+dF}{f_2} = +\frac{2f.\frac{3}{2}f}{F} = +3f$$

The second principal point is at a distance β from the eye lens,

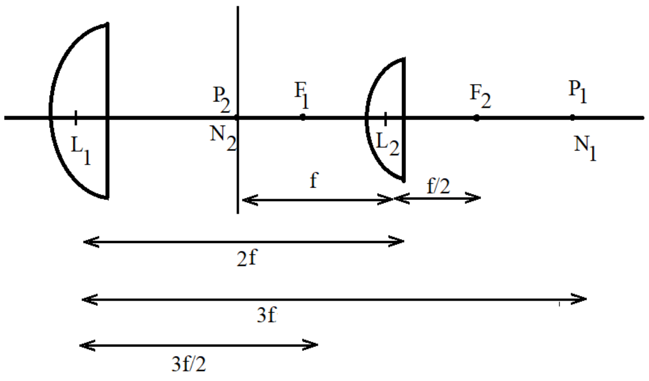
$$\beta = \frac{-dF}{f_1} = -\frac{2f.\frac{3}{2}f}{3f} = -f$$

The first focal point F_1 is at a distance $=f_1$-f = 3f-3f/2 = 3f/2 to the right of the field lens.

The second focal point F_2 is at distance = f-f_2 = 3f/2-f = f/2 = 0.5F to the left of the eye lens.

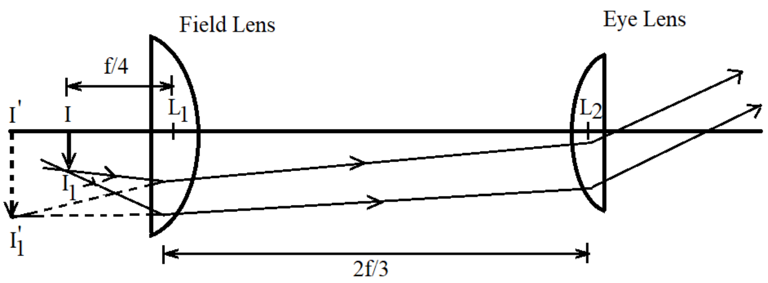
As the system is in air, the nodal points N_1 and N_2 coincide with the principal points P_1 and P_2.

The position of cardinal points is as shown in figure.



4. Ramsden Eyepiece:

It consists of two plano-convex lenses of equal focal length separated by the distance equal to two-thirds the focal length of either. The convex faces are towards each other and eyepiece is placed beyond the image formed by the objective as shown in fig.



In this eyepiece cross wires provided and it is used in optical instruments where accurate quantitative measurements are made.

Let F be the focal length of the equivalent lens.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} - \frac{\frac{2}{3}f}{f^2} = \frac{2}{f} - \frac{2}{3f} = \frac{6-2}{3f} = \frac{4}{3f}$$

$$\therefore F = \frac{3f}{4}$$

The equivalent lens must be placed at a distance 3f/4 behind the field lens at a distance α from it.

$$\alpha = \frac{dF}{f_2} = \frac{\frac{2f}{3} \times \frac{3f}{4}}{f} = \frac{\frac{f^2}{2}}{f} = \frac{f}{2}$$

Thus the equivalent lens is in between the field lens and eye lens.

As the focal length of the eyepiece (equivalent lens) is 3f/4, the image of the object due to the objective must be formed at a distance $\frac{3f}{4} - \frac{f}{2} = \frac{f}{4}$ in front of the field lens. This image will act as an object for the eyepiece and final image will be formed at infinity. The wires must be placed at the position where the image due to the objective is formed i.e., at a distance of f/4 in front of the field lens. This is the advantage of Ramsden eyepiece over the Huygens eyepiece.

Ramsden eyepiece is a positive eyepiece. The chromatic aberration in a Ramsden eyepiece is small. In some cases, both the lenses of the eyepiece are made of a combination of crown and flint glass and chromatic aberration is eliminated. As both the lenses are planoconvex with their convex surfaces facing each other the spherical aberration produced is small.

Cardinal Points of a Ramsden Eyepiece:

Ramsden constructed an eyepiece consisting of two plano-convex lenses of equal focal length separated by the distance equal to two-thirds the focal length of either.

Therefore, $f_1 = f_2 = f$ and d= 2f/3.

The equivalent focal length of the combination is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} - \frac{\frac{2}{3}f}{f^2} = \frac{2}{f} - \frac{2}{3f} = \frac{6-2}{3f} = \frac{4}{3f}$$

$$\therefore F = \frac{3f}{4}$$

The first principal point is at a distance α from the field lens,

$$\alpha = \frac{dF}{f_2} = \frac{\frac{2f}{3} \times \frac{3f}{4}}{f} = \frac{\frac{f^2}{2}}{f} = \frac{f}{2}$$

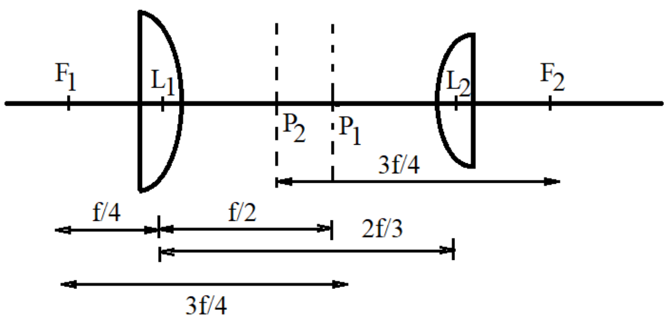
The second principal point is at a distance β from the eye lens,

$$\beta = \frac{-dF}{f_1} = -\frac{\frac{2f}{3} \times \frac{3f}{4}}{f} = -\frac{\frac{f^2}{2}}{f} = -\frac{f}{2}$$

Therefore, the first principal point P_1 is at a distance f/2 to right of the field lens and the second principal point P_2 is at a distance f/2 to left of the eye lens. The first focal point F1 is at a distance of 3f/4 from the first principal point and the second focal point F_2 is at a distance of 3f/4 from the second principal point.

Therefore, F_1 is at a distance $= \frac{3f}{4} - \frac{f}{2} = \frac{f}{4}$ to the left of the field lens and F_2 is at a distance of $\frac{3f}{4} - \frac{f}{2} = \frac{f}{4}$ to the right of the eye lens.

As the system is in air, the nodal points N_1 and N_2 coincide with the principal points P_1 and P_2. The position of cardinal points is as shown in figure.



Multiple choice questions:

1.There are __________cardinal points in an optical system

(a) two b) four **(c) six** (d) eight

2. The first principal focal points corresponds to the image distance is at -------

a) finite **b) infinite** c) origin d) both a and b

3. The second principal focal points corresponds to the ------distance is at infinity.

a) object b) image c) both a and b d) none of these

4. The nodal points are a pair of conjugate points on the axis having unit __________ angular magnification

(a) negative **(b) positive** (c) unequal (d) zero

5. The distance between two nodal points and the distance between two principal points are---

(a) equal (b) unequal (c) higher (d) lower

6. There are------ principal points in an optical system.

(a) two b) four (c) six (d) eight

7. For same medium, the principal points and nodal points are ------.

a) at different position **b) coincide to each other**

c) Both have not fixed position d) none of these

8. The distance of first principal plane from the first lens is

a)$\beta = -\frac{d\,f}{f_1}$ **b)** $\boldsymbol{\alpha = +\frac{df}{f_2}}$ c) $\beta = +\frac{d\,f}{f_2}$ d) $\alpha = +\frac{df}{f_1}$

9. The distance of second principal plane from the second lens is

a) $\boldsymbol{\beta = -\frac{d\,f}{f_1}}$ b)$\alpha = +\frac{df}{f_2}$ c) $\beta = +\frac{d\,f}{f_2}$ d) $\alpha = +\frac{df}{f_1}$

10. A co-axial lens system placed in air has two lenses of focal lengths 3F and F separated by a distance 2F, the equivalent focal length is-----------

a) 5F/2 **b) 3F/2** c) 2F d) 3F

11. The unit of power of lens is

a) diopters b) newton c) dyne d) pascal

12.What is the ratio of the focal lengths of the two lenses in Huygen's eyepiece?

a) 2:1 **b) 3:1** c) 3:2 d) 4:3

13. Huygens eyepiece is also known as ----------------

a) Spherical eyepiece b) Positive eyepiece

c) Negative eyepiece d) Double eyepiece

14. In Ramsden eyepiece two plano-convex lenses of ------ focal lengths are used.

a) equal b) different c) double d) one-third

15. The first principal point in Ramsden eyepiece is at ------- distance from the field lens

a) +3f/2 b) +2f/3 **c) +f/2** d)- f/4

16. The first principal point in Huygen's eyepiece is at ------- distance from the field lens

a) +3f b) -3f c) f d) 2f

17. In Huygens eyepiece, distance between two lenses is equal to the ------of their focal lengths.

a) sum **b) difference** c) multiplication d) division

18. Ramsden eyepiece is also known as ----------------

a) Spherical eyepiece **b) Positive eyepiece**

c) Negative eyepiece d) Double eyepiece

19. In Ramsden eyepiece distance between two plano-convex lenses is equal to -------the focal length of either.

a) one-third b) three-fourth **c) two-third** d) one-half

20. Huygens constructed an eyepiece consisting of two -------lenses.

a) plano-convex b) concave c) convex d) concave-convex